MATH 101 **FINAL EXAM Review**

College Algebra

The Final Exam is cumulative. Problems will be similar to those you have seen on previous exams. The exam is worth 120 points. Formulas provided on Exam 4 will also be provided on the Final.

Part I. Multiple Choice. The exam will have 15 multiple choice questions which will cover concepts, definitions, and symbolism. The questions will be similar to the MC questions on previous exams.
Part II. The remaining questions will be graded on partial credit. Graphs should be clearly labeled.

Some of the vocabulary and skills to review are included in the following table:

Vocabulary (concepts)		Skills (some)
function	logarithm	long division
zero of a function	exponential	synthetic division
domain	difference quotient	interpreting a graph
range	slope	factoring
matrix	transformations	factoring a polynomial
row echelon form	composition	quadratic formula
discriminant	piecewise function	u-substitution
asymptote	inverse function	squaring a binomial
rational function	polynomial function	composing functions
systems of equations:	end behavior	interval notation
- dependent	increasing/decreasing/constant	sign test for inequalities
- independent	maximum/minimum	
- consistent	rate of change	
- inconsistent	basic function families	

1. $f(x) = x^2 - 2x \qquad g(x) = \dfrac{1}{x-1} \qquad h(x) = 3x + 5$

   (a) Find and simplify $f(-2)$

   (b) Find and simplify $(g \circ h)(x)$

   (c) Find and simplify (i) $(f - h)(x)$ (ii) $(f - h)(-1)$

   (d) Simplify the difference quotient: $\dfrac{f(x+h) - f(x)}{h}$

2. Use the tables for $f(x)$ and $g(x)$ to find:

x	1	3	5	7
$f(x)$	5	7	1	3

(a) $(f \circ g)(3) =$ _____

(b) $(g \circ f)(3) =$ _____

x	1	3	5	7
$g(x)$	3	5	7	8

3. Write the equation of the line through the points (3, 7) and (-3, 4). Give your answer in slope-intercept form.

4. Write the equation of the line through (-4,5) that is perpendicular to the line $2x + 3y + 6 = 0$. Give your answer in slope-intercept form.

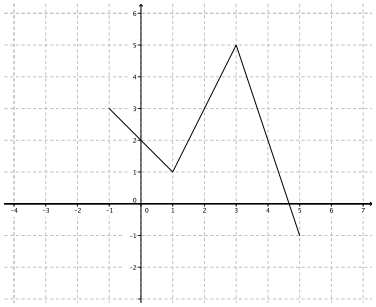
5. Complete the square to find the center and radius of the circle defined by $x^2 + y^2 + 14x - 4y - 19 = 0$

6. Describe each transformation. Use the graph of $y = f(x)$ given to sketch each transformation.

   (a) $y = f(x - 1) + 2$

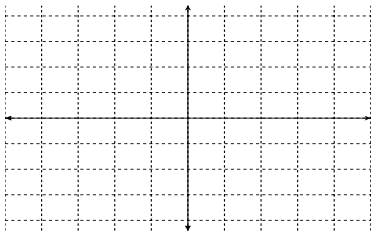
   (b) $y = \frac{1}{2} f(x)$

   (c) $y = -f(x)$

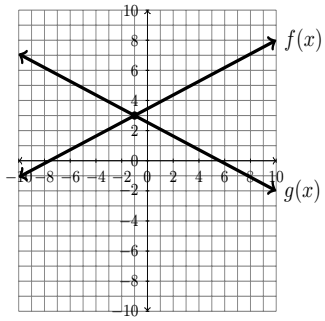


7. Graph the piecewise function:

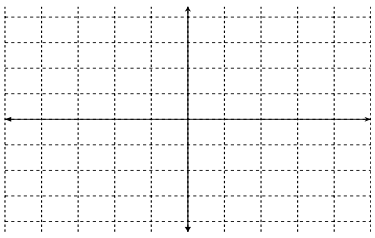
$$f(x) = \begin{cases} (x+2)^2 & -3 \le x < 0 \\ -2x & 0 \le x < 2 \end{cases}$$



8. Use the graph to solve $f(x) < g(x)$



9. Graph the function $f(x) = x^2 + 2x - 6$. [Label the scale clearly.]



(a) What is the vertex?

(b) State the domain and range.

(c) Write the equation for the axis of symmetry.

(d) Over what interval(s) is $f(x)$ increasing?

(e) Over what interval(s) is $f(x)$ decreasing?

(f) Find the real zero(s) for $f(x)$.

10. A cell phone company develops a pay-as-you-go cell phone plan in which the monthly cost varies directly as the number of minutes used. If the company charges \$17.70 in a month when 236 minutes are used, what is the constant of variation? What should it charge for a month in which 500 minutes are used?

11. Demand for a product varies inversely with the price per unit of the product. Demand for the product is 10,000 units when the price is \$5.75 per unit. What is the constant of variation? What is the demand (to the nearest hundred units) when the price is \$6.50?

12. The profit that a vendor makes per day by selling x pretzels is given by the function $P(x) = -0.002x^2 + 2.8x - 200$. Find the number of pretzels that must be sold to maximize profit.

13. The height of a model rocket launched from a 112 foot cliff is given by $h(t) = -16t^2+96t+112$, where t is the time, in seconds, from when the rocket is launched. Find the maximum height the rocket attains *above the cliff*.

14. Decide whether $(x + 4)$ is a factor of $P(x) = 2x^3 + 14x^2 + 23x - 4$.

15. Describe the end behavior of the graph of $f(x) = -x^4 + 2x + 5$

16. For the polynomial function $P(x) = (x^2 - 3)(x - 2)(x + 1)^2$, list all the zeros and their multiplicities. Determine whether the graph of $P(x)$ touches or crosses at each zero.

17. Find all the real or complex zeros of $f(x) = 3x^3 + x^2 - 34x + 40$ and write $P(x)$ in factored form.

18. Use the Rational Zeros Theorem to list the *possible* rational zeros for $f(x) = 10x^3 - 13x^2 - 103x - 20$.

19. Give the equations for any vertical, horizontal, or oblique asymptotes of
    (a) $f(x) = \dfrac{x^2 - x - 2}{2x^2 - x - 21}$ (b) $g(x) = \dfrac{3x^2 - 4x + 5}{x - 2}$ (c) $h(x) = \dfrac{2x + 3}{x^2 - 2x - 3}$

20. Sketch a graph of $f(x) = \dfrac{2x - 3}{x + 1}$. Label all intercepts and asymptotes.

21. Solve each inequality analytically (sign test). Verify graphically.
    (a) $\dfrac{(x - 2)(x + 1)}{(x + 4)} \leq 0$ (b) $x^2 - 5x + 4 < 0$

22. Find a formula for the inverse of $f(x) = \sqrt{x} - 3$
    (a) What is the domain for $f(x)$?
    (b) What is the domain for $f^{-1}(x)$?

23.
    (a) Write in logarithmic form: $2^{-3} = 0.125$
    (b) Evaluate: $\log_7 28$
    (c) Evaluate: $\ln e^{3.6}$

24. Solve for x: $\log_2(6x - 4) = 4.1$

25. Solve for x: $4^{2x-5} = \frac{1}{64}$

26. Solve for x: $2^{x-3} = 7^{2x}$

27. Solve for x: $e^{5x-1} = 4$

28. Solve for x: $3x^2 - 2x + 11 = 0$

29. Solve for x: $|4x - 3| = 9$

30. Solve for x: (a) $|2x + 8| \leq 4$ (b) $|7 - 2x| + 3 > 11$

31. Solve for x:

32. Use the properties of logarithms to express the following as a sum, difference, or product of simpler logarithms: $\log \dfrac{m^3 n}{\sqrt{y}}$

33. Suppose \$15,000 is invested at 3.25%. Find the total amount present at the end of 5 years if interest is compounded

    (a) quarterly

    (b) continuously

34. If Tim has \$1000 to invest at 7% compounded continuously, how long will it be before he has \$1500? How long before his investment is doubled?

35. If Seattle had a population of 2.3 million in 1990 and 1.75 million in 1950, use the continuous growth model $A(t) = A_0 e^{kt}$ to find the rate of growth and predict the population for 2015.

36. Assume fish populations obey the exponential growth model ($A(t) = A_0 e^{kt}$). There are 15 angel fish in a fish tank at the beginning of the month. Thirty days later there are 55 angel fish.

    (a) Find an exact representation for the rate of growth, k.

    (b) How many fish will there be in the tank in 2 weeks? (note the units)

    (c) Use the exact value to determine how many days will it be before there are 100 angel fish?

37. The radioactive isotope of potassium ^{42}K which is used in the diagnosis of brain tumors, has a half-life of 12.4 hours. If 500 milligrams are taken, how many milligrams will remain after 24 hours?

38. A sheet of heavy-duty cardboard measuring 40 inches by 30 inches is to be made into an open box by cutting out equal-sized squares of side length x from each corner and folding up the sides.
    a) What are the restrictions on the values for x?
    b) Write a function V, which will give the volume of the open box.
    c) Find the volume if a square that is 4.5 inches on a side is cut out.
    d) For what value of x will the volume be a maximum? What is the maximum volume?

39. Solve the following linear systems by substitution, elimination, or matrix row operations.

(a) $\begin{cases} 6x + 3y = 12 \\ 2x - y = -2 \end{cases}$

(b) $\begin{cases} 2x + 3y = 27 \\ y - x = -1 \end{cases}$

(c) $\begin{cases} \frac{1}{3}x - 2y = 1 \\ 5x - 30y = 18 \end{cases}$

(d) $\begin{cases} x + 2y + 4z = -3 \\ 2x + 7y + 15z = -12 \\ 4x + 7y + 13z = -10 \end{cases}$

40. The given matrix represents a linear system. Use matrix row operations to solve, and write the solution as an ordered triple.

$$\left[\begin{array}{ccc|c} 1 & 4 & -3 & 8 \\ 0 & -2 & 1 & 6 \\ 0 & 4 & -2 & -12 \end{array}\right]$$

41. Write the augmented matrix for the linear system below. Then, perform the indicated row operations (in order) to transform the matrix.

$$\begin{cases} 2x - y + 3z = 0 \\ x + 2y - z = 5 \\ 2y + z = 1 \end{cases}$$

a) $R_1 \leftrightarrow R_2$
b) $2R_2 + R_3 \longrightarrow R_3$

42. Clearly define the variables and set up a system of equations to solve. Then solve by substitution, elimination, or matrix row operations.
An inheritance of $5,000 is divided among three investments yielding $240 in simple interest per year. The interest rates for the investments are 3%, 4%, and 6%. The difference in the amount of money invested in the fund paying 6% and the amount invested for the least return is the same as the amount invested at 4%. Find the amount of each investment.

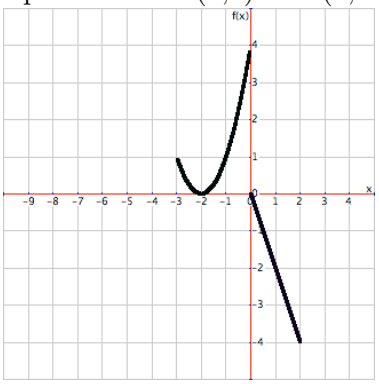
43. Solve the following non-linear system algebraically and graphically.

$$\begin{cases} (x-2)^2 + (y+1)^2 = 9 \\ y = -x + 4 \end{cases}$$

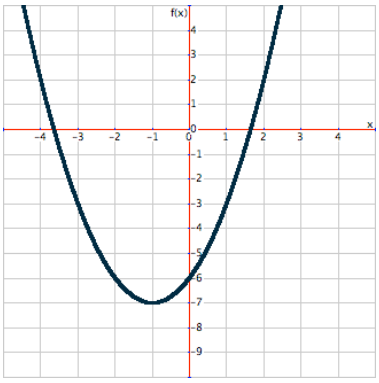
44. Complete the square to find the center and radius of the following equation of a circle:
$x^2 + 6x + y^2 - 10y - 15 = 0$.

Answers:

1. (a) 8 (b) $\dfrac{1}{3x+4}$ (c) (i) x^2-5x-5 (ii) 1 (d) $2x+h-2$
2. (a) 1 (b) 8
3. $y=\frac{1}{2}x+\frac{11}{2}$
4. $y=\frac{3}{2}x+11$
5. $C:(-7,2)$, $r=6\sqrt{2}$
6. (a) The new graph is shifted 1 unit right and 2 units up. Key points: (0, 5), (2, 3), (4, 7), (6, 1)
   (b) The new graph is a vertical shrink by a factor of $\frac{1}{2}$. Key points: (-1, 1.5), (1, .5), (3, 2.5), (5, -.5)
   (c) The new graph is reflected over the x-axis. Key points: (-1, -3), (1, -1), (3, -5), (5, 1)
7. open circle on (0,4) and (2,-4)



8. $(-\infty,-1)$
9.



(a) (-1, -7)

(b) D: $(-\infty,\infty)$; R: $[-7,\infty)$

(c) $x=-1$

(d) $[-1,\infty)$

(e) $(-\infty,-1]$

(f) $x=-1\pm\sqrt{7}$

10. $k = \$.075$ charge per minute; \$37.50

11. $k = 57,500$; ≈ 8800 units

12. 700 pretzels

13. maximum height above the cliff: 144 feet

14. yes; $P(-4) = 0$

15. falls on the left (as $x \to -\infty$, $f(x) \to -\infty$); falls on the right (as $x \to \infty$, $f(x) \to -\infty$)

16. crosses at $x = \pm\sqrt{3}, 2$ (each multiplicity 1) and touches at $x = -1$ (multiplicity 2)

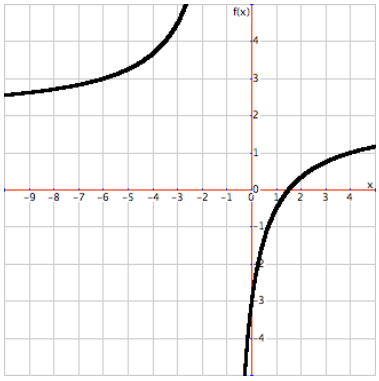
17. $f(x) = 3(x-2)(x+4)(x-\frac{5}{3})$ or $f(x) = (x-2)(x+4)(3x-5)$; zeros are 2, -4, and $\frac{5}{3}$

18. $\pm\{1, 2, 4, 5, 10, 20, \frac{1}{2}, \frac{1}{5}, \frac{1}{10}, \frac{2}{5}, \frac{4}{5}, \frac{5}{2}\}$

19.

(a)	(b)	(c)
VA: $x = \frac{7}{2}$, $x = -3$	VA: $x = 2$	VA: $x = 3, x = -1$
HA: $y = \frac{1}{2}$	HA: none	HA: $y = 0$
OA: none	OA: $y = 3x + 2$	OA: none

20.



VA: $x = -1$
HA: $y = 2$
y-int: (0, -3)
x-int: $(\frac{3}{2}, 0)$

21.

	(a)	(b)
critical values:	2, -1, -4	1, 4
test intervals:	$(-\infty, -4), (-4, -1), (-1, 2), (2, \infty)$	$(-\infty, 1), (1, 4), (4, \infty)$
work:	*show tests*	
answer:	$(-\infty, -4) \cup [-1, 2]$	$(1, 4)$

22. $f^{-1}(x) = (x+3)^2, x \geq -3$ (a) Domain$_f$: $[0, \infty)$ (b) Domain$_{f^{-1}}$: $[-3, \infty)$

23. (a) $log_2\ 0.125 = -3$ (b) 1.712 (c) 3.6

24. $x \approx 3.525$

25. $x = 1$

26. $x = \dfrac{3\,log2}{log2 - 2\,log7} \approx -0.65$

27. $x = \dfrac{ln4 + 1}{5} \approx .477$

28. $x = \dfrac{1 \pm 4i\sqrt{2}}{3}$

29. $x = 3$ and $x = -\frac{3}{2}$

30. $[-6, -2]$

31. $(-\infty, \frac{1}{2}) \ \bigcup \ (\frac{15}{2}, \infty)$

32. $3\,log\,m + log\,n - \left(\frac{1}{2}\right) log\,y$

33. (a) \$17,635.14 (b) \$17,646.72

34. 5.79 years; 9.9 years

35. $k = \dfrac{ln\left(\frac{2.3}{1.75}\right)}{40} \approx 0.0068$; 2.73 million people in 2015

36. (a) $k = \dfrac{ln(\frac{55}{15})}{30} \approx 4.33\%$; (b) $\approx 27.5 \longrightarrow$ 27 fish; (c) 43.8 days

37. $k = \dfrac{ln(.5)}{12.4} \approx -.0559$; 130.7 milligrams remain after 24 hours

38. a) $0 < x < 15$ b) $V(x) = (40 - 2x)(30 - 2x)x = 1200x - 140x^2 + 4x^3$ c) 2929.5 cu.in.
d) $x \approx 5.7in.$, $V(5.7) \approx 3032.2$ cu.in.

39.
(a) $(\frac{1}{2}, 3)$ ((b) (6, 5)
(c) $\emptyset$ (d) (1, -2, 0)

40. $(z + 20, \frac{1}{2}z - 3, z)$

41.
augmented matrix (a) (b)

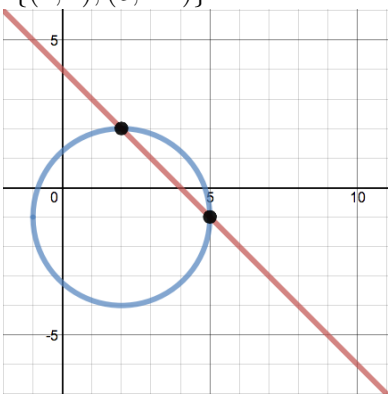
$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 0 \\ 1 & 2 & -1 & 5 \\ 0 & 2 & 1 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 2 & -1 & 3 & 0 \\ 0 & 2 & 1 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 2 & -1 & 3 & 0 \\ 4 & 0 & 7 & 1 \end{array}\right]$$

42.
$x =$ dollars invested at 3%
$y =$dollars invested at 4%
$z =$ dollars invested at 6%

$$\begin{cases} x + y + z = 5000 \\ .03x + .04y + .06z = 240 \\ z - x = y \end{cases}$$

$x = \$1000$, $y = \$1500$, $z = \$2500$

43. $\{(2,2),(5,-1)\}$



44. Center: $(-3,5)$; Radius $= 7$